

# The Learning Activity

## The Problem

### The Need

One year after the completion of the new Grant Road Bridge, the Hauptville Town Engineer inspects the structure and finds that it is performing well. Though the bridge has been carrying a lot of traffic, its structural members show no signs of distress or deterioration. Nonetheless, the Town Engineer is still somewhat concerned about the bridge. Because of a major construction project nearby, many heavily loaded dump trucks have been using Grant Road recently. What if one of these trucks is heavier than the legal weight limit? How much of an overload would cause the structure to collapse? The Town Engineer decides to perform a complete structural evaluation to determine the overall level of safety of the Grant Road Bridge. He begins by hiring Universal Structural Materials Assessment, Inc. to test the strength of the structural members used in the bridge. (We did this part of the structural evaluation in Learning Activity #2.) Once the Engineer has received the test results from Universal, he is ready to begin his analysis.

### Your Job

You are the Town Engineer of Hauptville. Your job is to analyze the Grant Road Bridge and evaluate its overall level of safety. Specifically, you must calculate the factor of safety for every member in one of the main trusses, then determine the overall safety factor for the structure.

As the Town Engineer, you have the professional responsibility to protect the health and safety of the people who use this bridge. You fulfill this responsibility by performing the structural evaluation conscientiously—by using good judgment, by performing calculations carefully and accurately, and by asking a colleague to check your work.

## The Solution

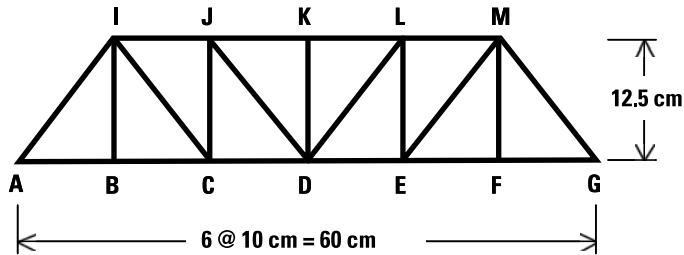
### The Plan

Our plan to conduct the structural analysis and evaluation of the Grant Road Bridge consists of the following tasks:

- Create the structural model.
- Check the structural model for static determinacy and stability.
- Calculate the reactions.
- Calculate the internal member forces.
- Determine the strengths of the members
- Calculate the factor of safety for every member in the structure
- Evaluate the safety of the structure.
- Check our assumptions.

## Create the Structural Model

To model the Grant Road Bridge, we must define (1) the geometry of the structure, (2) the loads, and (3) the supports and reactions. We begin by idealizing the three-dimensional bridge structure as a pair of two-dimensional Pratt trusses. Since these two trusses are identical, we only need to analyze one of them. The geometry of the truss is shown below. The dimensions indicate the locations of the *centerlines* of the members. Joints are identified with letters—the same letter designations that were used on the bridge plans provided in Learning Activity #1. To facilitate the analysis, we will assume that the truss members are perfectly straight, the joints are frictionless pins, and the loads are applied only at the joints. We will also assume that the weight of the truss itself is zero.



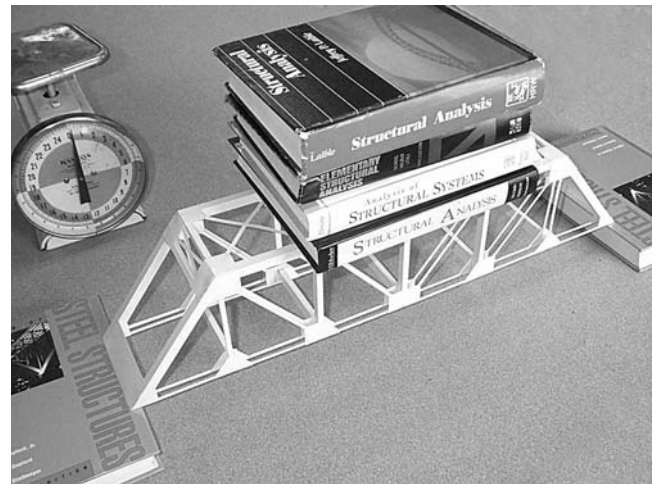
# Q1

Why did we assume that the weight of the truss is zero?

Obviously, the actual weight of the truss is not zero. Why did we make this assumption, when we know it is not true? How do you think it will affect the accuracy of our structural analysis?

When we load-tested the Grant Road Bridge in Learning Activity #1, we applied the load in two different ways—with a stack of books placed on the top chord and with a bucket of sand suspended from the floor beams. Before we can define the loads for our structural model, we need to decide which of these two loading configurations to use. As a general rule, a structural evaluation should be based on the most severe loading condition—the one that produces the highest member forces. If the analysis shows that the truss is safe for the most severe loading, then the structure will certainly be safe for less severe ones. Unfortunately, in this case, it is not immediately obvious which of the two loading configurations is more severe. The best we can do is to make an assumption and check it later. For now, we will assume that the top-chord loading, shown here, is more severe.

Having decided on the location of the load, we must now determine its magnitude. In Learning Activity #2, we applied the equation  $W=mg$  to determine that the weight of a 5-kilogram mass is 49.05 newtons. When we placed the stack of books onto the top chord of the truss, the weight of the stack was supported on joints J, J', K, K', L, and L'. We can reasonably assume that the weight of the books is distributed equally to these six joints. Therefore, the downward force applied to each joint is

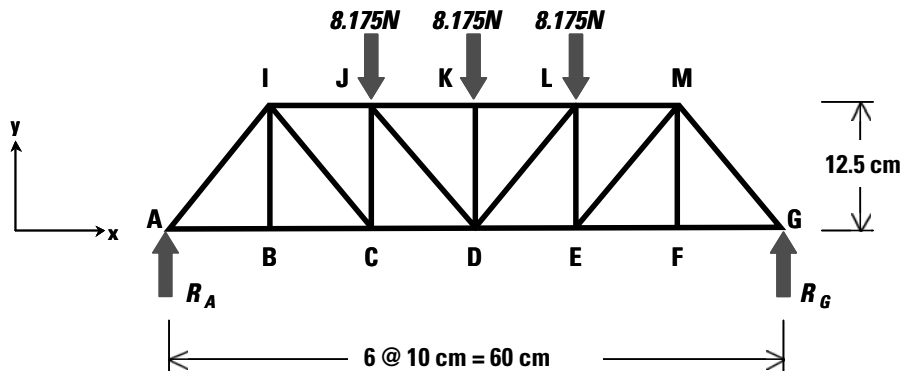


We will assume that the top chord loading is more severe, then check the bottom-chord loading later.

$$\text{Load per Joint} = \frac{\text{Total Load}}{\text{Number of Joints}} = \frac{49.05\text{N}}{6} = 8.175\text{N}$$

Since there are two main trusses, three **8.175N** loads will be applied to each truss. Since all of the loads are downward, and the bridge is supported only at its ends, we will add upward reactions  $R_A$  and  $R_G$  at joints A and G.

A complete free body diagram of the truss looks like this:



## Check Static Determinacy and Stability

Before we can use the equations of equilibrium to analyze this truss, we must first verify that it is statically determinate and stable. The mathematical condition for static determinacy and stability is

$$2j = m + 3$$

where  $j$  is the number of joints and  $m$  is the number of members. Our truss from the Grant Road Bridge has 12 joints and 21 members. Substituting these numbers into the equation above, we find that  $2j$  and  $m+3$  are both equal to 24, so the mathematical condition for static determinacy and stability is satisfied. Furthermore, we note that the truss is composed entirely of interconnected triangles, which confirms our conclusion that the structure is stable.

## Calculate Reactions

On the free body diagram above, the forces  $R_A$  and  $R_G$  are the unknown reactions at Joints A and G. We know that the truss is in equilibrium; therefore, the sum of all forces acting on the structure must be zero. Since all of the forces—loads and reactions—are acting in the  $y$ -direction, only one of our two equilibrium equations is relevant to the calculation of reactions:

$$\sum F_y = 0$$

$$R_A + R_G - 8.175 - 8.175 - 8.175 = 0$$

Since the structure, the loads, and the reactions are all symmetrical about the centerline of the truss, the two reactions  $R_A$  and  $R_G$  must be equal. (The centerline of the truss is a vertical line passing through Member DK). Substituting  $R_A = R_G$  into the equilibrium equation above, we get

$$R_A + R_A - 24.525 = 0$$

$$2R_A = 24.525$$

$$R_A = 12.26\text{N} \uparrow$$

And since  $R_A = R_G$ , then

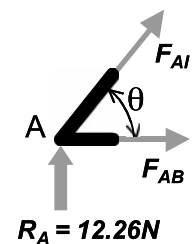
$$R_G = 12.26\text{N} \uparrow$$

## Calculate Internal Member Forces

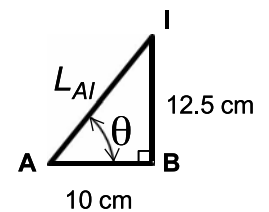
We will use the Method of Joints to calculate the internal force in each member of the truss. To apply this method, we will isolate a joint from the structure, cutting through the attached members and exposing their internal member forces. We will draw a free body diagram of the joint, then use the equations of equilibrium to determine the unknown member forces. We will repeat the process for successive joints, until we have calculated all of the internal member forces in the structure.

### Joint A

We'll start by isolating Joint A and drawing a free body diagram of it. The free body diagram must show *all* forces acting on the joint. Thus the reaction  $R_A$  is shown, along with its known magnitude of 12.26N. The member forces  $F_{AI}$  and  $F_{AB}$  are also included on the diagram. Because we do not know the magnitudes or directions of these forces, we simply show them in variable form, and we assume their directions to be in tension. To indicate that a member force is in tension, we draw the force vector pointing away from the joint, along the centerline of the member.



Before we can write the equilibrium equations for this joint, we need to figure out what the angle  $\theta$  is. Actually, we don't really need to know the angle itself; rather, we only really need to know the sine and cosine of the angle— $\sin\theta$  and  $\cos\theta$ . We can determine the sine and cosine directly from the geometry of the truss. Note that Members AB, AI, and BI form a right triangle, with Member AI as the hypotenuse. We can apply the Pythagorean Theorem to calculate the length,  $L_{AI}$ , as follows:



$$L_{AI} = \sqrt{10^2 + 12.5^2} = 16.01\text{cm}$$

Now we can apply the basic definitions of the sine and cosine to find  $\sin\theta$  and  $\cos\theta$ :

$$\sin\theta = \frac{\textit{opposite}}{\textit{hypotenuse}} = \frac{12.5}{16.01} = 0.7809$$

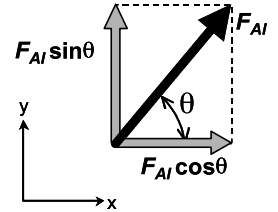
$$\cos\theta = \frac{\textit{adjacent}}{\textit{hypotenuse}} = \frac{10}{16.01} = 0.6247$$

We are finally ready to write the equilibrium equations for Joint A. We will start with the equation for the sum of forces in the y-direction. Assuming that the upward direction is positive,

$$\sum F_y = 0$$

$$12.26 + F_{AI} \sin\theta = 0$$

To write this equation, we had to represent the force  $F_{AI}$  in terms of its x-component and y-component. The y-component is  $F_{AI}\sin\theta$ , and its direction is upward, so it is positive in the equilibrium equation. The x-component is  $F_{AI}\cos\theta$ ; however, this component is not included in the  $\Sigma F_y$  equilibrium equation, because it does not act in the y-direction.



We can now substitute the known value of  $\sin\theta$  into the equilibrium equation, and solve for the unknown force  $F_{AI}$ .

$$12.26 + F_{AI}(0.7809) = 0$$

$$F_{AI}(0.7809) = -12.26$$

$$F_{AI} = -\frac{12.26}{0.7809} = -15.70N$$

Because the answer is negative, our initial assumption about the direction of  $F_{AI}$  was incorrect. We assumed that  $F_{AI}$  is in tension. The negative member force indicates that it is in compression. Thus our final answer is

$$F_{AI} = \underline{\underline{15.70N \text{ (compression)}}}$$

Now we can write the equilibrium equation for forces in the x-direction. Assuming that the positive direction is to the right,

$$\sum F_x = 0$$

$$F_{AB} + F_{AI} \cos\theta = 0$$

We know  $\cos\theta$ , and we have just solved for  $F_{AI}$ . We can substitute these values into the equilibrium equation and solve for  $F_{AB}$ . But be careful! When you substitute  $F_{AI}$ , don't forget the minus sign.

$$F_{AB} + (-15.70)(0.6247) = 0$$

$$F_{AB} = +9.81N$$

Because the answer is positive, our assumption about the direction of  $F_{AB}$  was correct. The final answer is

$$F_{AB} = \underline{\underline{9.81N \text{ (tension)}}}$$

# Q<sub>2</sub>

Why did we choose to do Joint A first?

Is there a reason why Joint A was a good place to start this analysis? What would have happened if we had started with a different joint?

# Q<sub>3</sub>

Why did we choose to solve the y-direction equilibrium equation first?

Is there a reason why it was a good idea to solve  $\sum F_y = 0$  before solving  $\sum F_x = 0$ ?

## Joint B

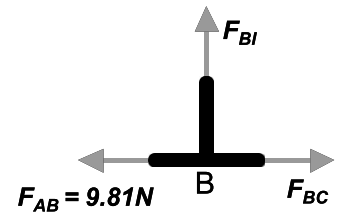
At this point, we should analyze Joint B. It has only three connected members, and we already know the internal force in one of the three (Member AB). Thus there are only two unknown forces, and we will be able to solve for them with the two available equilibrium equations.

Again we draw a free body diagram of the joint, with all member forces assumed to be in tension—pointing away from the joint. The known magnitude of the force  $F_{AB}$  is included on the diagram. The equilibrium equation for forces in the x-direction is

$$\sum F_x = 0$$

$$-9.81 + F_{BC} = 0$$

$$F_{BC} = +9.81\text{N} = \underline{\underline{9.81\text{N (tension)}}}$$



The equilibrium equation for forces in the y-direction produces an interesting result:

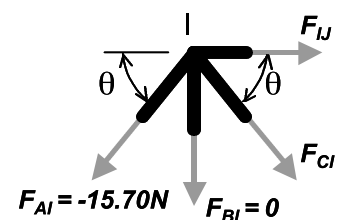
$$\sum F_y = 0$$

$$F_{BI} = 0$$

It should come as no surprise that this member has zero internal force. When you load-tested the Grant Road Bridge in Learning Activity #1, you should have noticed that this member—the hip vertical—was slack. It appeared to have no internal force at all. Now we have verified our observation using the Method of Joints!

## Joint I

It takes some careful thought to recognize that Joint I should be the next joint we analyze. As the free body diagram indicates, this joint has four connected members and, therefore, it also has four internal member forces. Note, however, that we have already calculated two of these— $F_{AI}$  and  $F_{BI}$ . Thus there are only two unknown forces, which we can calculate with our two equilibrium equations.



Note that all four of the force vectors are pointing away from the joint, even though we already know that one of them,  $F_{AI}$  is in compression. The negative magnitude of  $F_{AI}$  ensures that it is mathematically represented as a compression force.

It is important to note that both angles labeled as  $\theta$  on the free body diagram are exactly the same as the angle  $\theta$  on the diagram of Joint A. (If you can't see that these angles are all equal, prove it to yourself by drawing the corresponding right triangles, just as we did for Joint A.) Thus the values we calculated for  $\sin\theta$  and  $\cos\theta$  for Joint A are still valid here.

If we begin with the equilibrium equation in the y-direction, we will be able to solve for  $F_{CI}$  directly:

$$\begin{aligned}\sum F_y &= 0 \\ -F_{AI} \sin\theta - F_{BI} - F_{CI} \sin\theta &= 0 \\ -(-15.70)(0.7809) - 0 - F_{CI}(0.7809) &= 0 \\ F_{CI} &= +15.70 = \underline{\underline{15.70N \text{ (tension)}}}\end{aligned}$$

Now we can use the second equilibrium equation to solve for  $F_{IJ}$ :

$$\begin{aligned}\sum F_x &= 0 \\ -F_{AI} \cos\theta + F_{CI} \cos\theta + F_{IJ} &= 0 \\ -(-15.70)(0.6247) + 15.70(0.6247) + F_{IJ} &= 0 \\ F_{IJ} &= -19.62N = \underline{\underline{19.62N \text{ (compression)}}}\end{aligned}$$

### Joint C

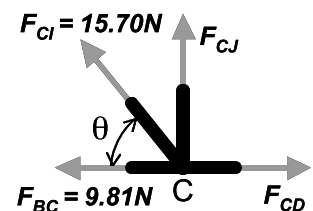
Next we will calculate the unknown member forces at Joint C.



#### Why Joint C?

Why was Joint C the best joint to analyze at this point in the solution process?

Based on the free body diagram of the joint, we can write the two equilibrium equations and solve for the two unknown member forces as follows:



$$\sum F_x = 0$$

$$-F_{BC} - F_{CI} \cos\theta + F_{CD} = 0$$

$$-9.81 - (15.70)(0.6247) + F_{CD} = 0$$

$$F_{CD} = +19.62N = \underline{\underline{19.62N \text{ (tension)}}}$$

$$\sum F_y = 0$$

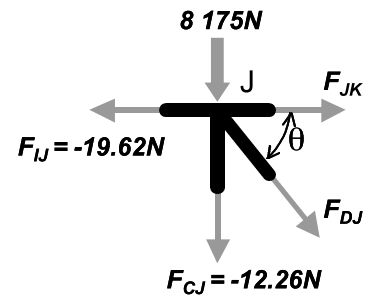
$$F_{CI} \sin\theta + F_{CJ} = 0$$

$$(15.70)(0.7809) + F_{CJ} = 0$$

$$F_{CJ} = -12.26N = \underline{\underline{12.26N \text{ (compression)}}}$$

### Joint J

The free body diagram of Joint J is shown at right. Note that the 8.175N load at Joint J *must* be included on the diagram. (Failure to put loads on the free body diagram is one of the most common errors in truss analysis.) We can write the two equilibrium equations and solve for the two unknown member forces as follows:



$$\sum F_y = 0$$

$$-8.175 - F_{CJ} - F_{DJ} \sin\theta = 0$$

$$-8.175 - (-12.26) - F_{DJ}(0.7809) = 0$$

$$F_{DJ}(0.7809) = 4.085$$

$$F_{DJ} = +5.23N = \underline{\underline{5.23N \text{ (tension)}}}$$

$$\sum F_x = 0$$

$$-F_{IJ} + F_{DJ} \cos\theta + F_{JK} = 0$$

$$-(-19.62) + (5.23)(0.6247) + F_{JK} = 0$$

$$F_{JK} = -22.89N = \underline{\underline{22.89N \text{ (compression)}}}$$

# Q5

## Can you apply the Method of Joints to calculate a member force?

Which joint should you analyze to determine the member force  $F_{DK}$ ? Solve the appropriate equilibrium equations to show that  $F_{DK}=8.175\text{N}$  (compression).

### Summary of Structural Analysis Results

At this point, we have only analyzed half of the truss. However, if we take advantage of symmetry, we can determine the internal forces in all remaining members without doing any further calculations. When we determined the reactions  $R_A$  and  $R_G$ , we noted that these two forces must be equal because the structure, its loads, and its reactions are all symmetrical about the centerline of the truss. The same principle holds true for internal member forces. Because the structure, loads, and reactions are all symmetrical, the member forces must also be symmetrical about the centerline. Members that are “mirror images” of each other have equal internal forces.  $F_{GM}$  and  $F_{AI}$  must be equal;  $F_{LM}$  and  $F_{IJ}$  must be equal;  $F_{DJ}$  and  $F_{DL}$  must be equal, and so forth.

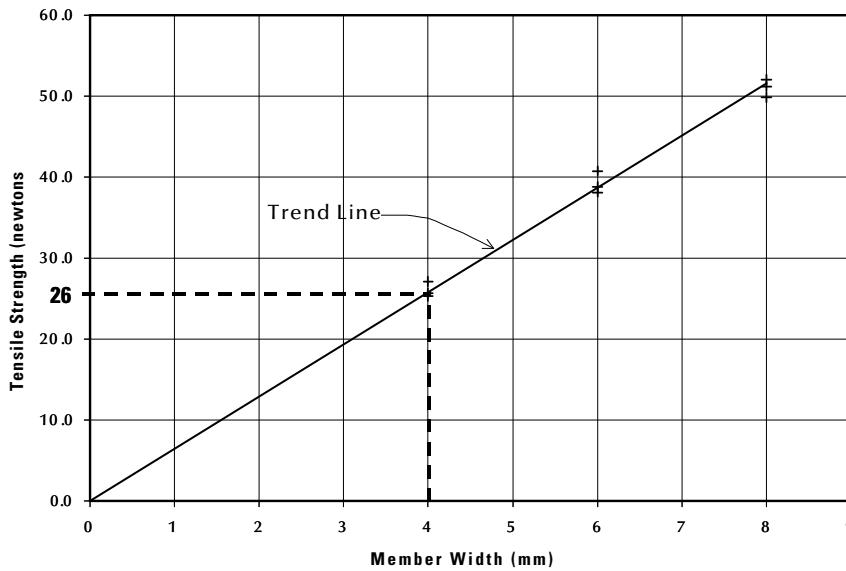
So we’re done! The results of the analysis are summarized in the table below.

Members	Force	Members	Force
AB, FG	9.81 N (tension)	BI, FM	0 N
BC, EF	9.81 N (tension)	CI, EM	15.70 N (tension)
CD, DE	19.62 N (tension)	CJ, EL	12.26 N (compression)
IJ, LM	19.62 N (compression)	DJ, DL	5.23 N (tension)
JK, KL	22.89 N (compression)	DK	8.175 N (compression)
AI, GM	15.70 N (compression)		

## Determine the Strengths of the Members

Now that we have calculated the force in each member, we must determine the corresponding strength of each member. To do this, we will use the graphs we developed in Learning Activity #2. We’ll start with the *bars*—the bottom chords, diagonals, and hip verticals. The table above tells us what we have already observed in our Grant Road Bridge model—that all of the bars are in tension (except the hip verticals, which have zero

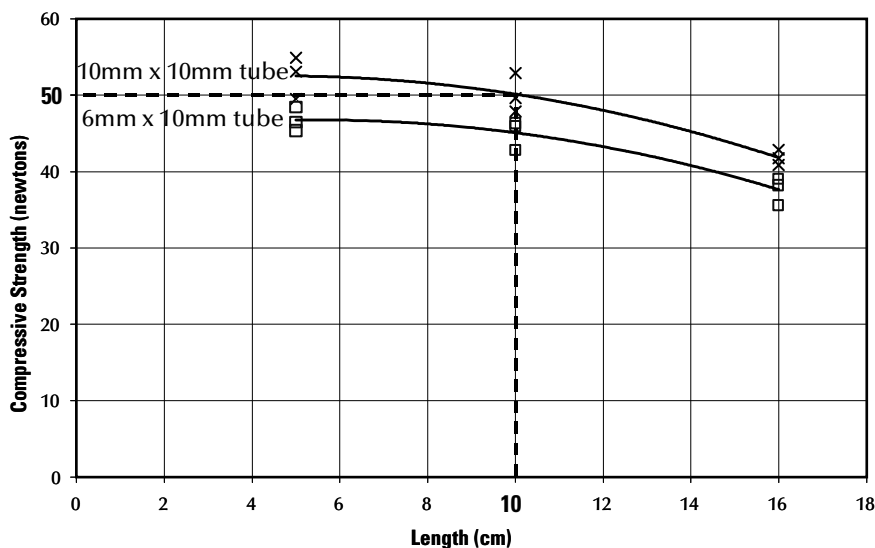
internal force). Thus we need to determine the *tensile strength* of the bars. All of the bars used in the Grant Road Bridge are 4mm wide. Using the tensile strength vs. member width graph we developed in Learning Activity #2, we find the tensile strength of a 4mm bar to be 26 newtons, as shown below.



**Determining the tensile strength of a 4mm bar**

Note, however, that all of the bottom chords, diagonals, and hip verticals are actually *doubled* 4mm bars. Thus the tensile strength of these members is exactly twice that of a single 4mm bar, or 52 newtons.

Our structural analysis shows that all of the tubes—the top chords, the end posts, and the interior verticals—are in compression. Thus we must determine the compressive strength of these members, using the strength vs. length graph we developed in Learning Activity #2. The top chord members are all 10mm x 10mm tubes, and each has a length of 10 centimeters. The strength of these members is approximately 50 newtons, as indicated below.



**Determining the compressive strength of a 10mm x 10mm tube that is 10 centimeters long**

# Q6

## Can you determine the strength of a member?

What is the compressive strength of the vertical tube members (CJ, DK, and EL) and the end posts (AI and GM)?

## Calculate the Factor of Safety

Once we know the strength of a member and the internal force it is actually experiencing, we can calculate its factor of safety. For the bottom chord member CD, the factor of safety is:

$$FS_{CD} = \frac{\text{Strength}}{\text{Internal Member Force}} = \frac{52}{19.62} = 2.7$$

For the top chord member JK, the factor of safety is

$$FS_{JK} = \frac{\text{Strength}}{\text{Internal Member Force}} = \frac{50}{22.89} = 2.2$$

# Q7

## Can you calculate the factor of safety for a member?

Calculate the factor of safety for all remaining members in the truss, and add them to the summary table below (along with the member strengths not already recorded in the table).

Members	Force	Strength	FS
AB, FG	9.81 N (tension)	52	
BC, EF	9.81 N (tension)	52	
CD, DE	19.62 N (tension)	52	2.7
IJ, LM	19.62 N (compression)	50	
JK, KL	22.89 N (compression)	50	2.2
AI, GM	15.70 N (compression)		
BI, FM	0 N --	52	--
CI, EM	15.70 N (tension)	52	
CJ, EL	12.26 N (compression)		
DJ, DL	5.23 N (tension)	52	
DK	8.175 N (compression)		

## Evaluate the Structure

As the Town Engineer of Hauptville, you have finished what you set out to do—a complete structural evaluation of the main trusses of the Grant Road Bridge. Yet the results of these calculations are just numbers. They are of little use, until you study them, think critically about them, and draw meaningful conclusions from them.

Once you have completed the summary table on the previous page, you should be able to make the following observations:

- Members JK and KL have a factor of safety of 2.2—the smallest of any member in the truss. Since the failure of Member JK or Member KL would cause the entire structure to collapse, we can say that the factor of safety of the *entire structure* is 2.2.
- Since 2.2 is obviously larger than 1, our analysis tells us that the structure will not collapse when the 5 kg mass is placed on the top chord. Because the factor of safety is considerably larger than 1, we can have a high degree of confidence that the structure will not fail, even if we made some minor errors in construction or if the actual load is significantly larger than 5 kg.
- Theoretically, the bridge would collapse if the mass of the stack of books at mid-span were increased to  **$(5\text{ kg})(2.2)=11.0\text{ kg}$** .
- Many members of the truss have safety factors that are substantially larger than 2.2. These members are actually much stronger than they need to be.



### Why are some truss members stronger than they need to be?

For example, Member CI is a doubled 4mm bar with a safety factor of 3.3. Had the structural engineer chosen to use a doubled 3mm bar for this member, the safety factor would still be 2.5. The member would use less material; and because its safety factor would still be greater than 2.2, the overall safety of the structure would not be adversely affected. Why did the structural engineer choose not to use a smaller member size?

It is important to note that this structural evaluation is valid only for one particular loading. If we change either the magnitude or the position of the load, the member forces and factors of safety will also change.